LBT PROJECT
2x8,4m TELESCOPE

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LBT PROJECT

2 X 8,4m OPTICAL TELESCOPE

Instrument Rotator and Cable Chain

Mechanical Design Calculations

	Signature	Date
Prepared	Robert Meeks	15-Mar-07
Reviewed		
Approved		

1.0 Revision History

Issue	Date	Changes	Responsible
a	15-Nov-07	First Draft	Robert Meeks

2.0 Table of Contents

3.0 About this Document

3.1 Purpose

The purpose of this document is to show the various calculations and analyses performed to support design decisions related to the mechanical design of the instrument rotator and cable chains and to document the source of any information used in those decisions.

3.2 Reference Documents

1. LBTO Document No. 671s001c, *Instrument Rotator and Cabel Chain Technical Specifications*.
2. LBTO Document No. 600s001a, *LBT f/15 Gregorian Focus Optical Design*
3. LBTO Document No. 600s003a, *LBT f/15 Bent Gregorian Focus Optical Design*
4. LBTO Document No. 671x001b, *Bent Gregorian Cable Chain Bearing and Gear Drawing*
5. LBTO Document No. 671x000b, *Bent Gregorian Rotator Bearing Drawing*
6. LBTO Document No. 674x008b, *Direct Gregorian Rotator Bearing Drawing*
7. LBTO Document No. 671x009b, *Direct Gregorian Cable Chain Bearing and Gear Drawing*
8. LBTO Document No. 671x002a, *Bent Gregorian Rotator Bearing Information*
9. LBTO Document No. 670s007a, *Detailed Design Description*
10. LBTO Document No. 671s005g, *Bent Gregorian Rotator Gear Drawing*
11. LBTO Document No. 674x004b, *Direct Gregorian Rotator Gear Drawing*
12. Kollmorgen data sheet
13. Shigley, J. E. and Mitchell, L. D., *Mechanical Engineering Design*. 4th ed.
14. LBTO Document No. 671s009a, *Instrument Rotator Motor Mount Finite Element Analysis Report*
15. LBTO Document No. 670s013a, *Instrument Rotator and Cable Chain Mechanical Component Data Package*
16. LBTO Document No. 670s006a, *Instrument Rotator and Cable Chain System Analysis*
17. Louis Carpasso, Farrand Controls, Personal communication, 1/3/2007
18. LBTO Document No. 670s014a, *Instrument Rotator and Cable Cable Chain Torque Estimate*

4.0 Requirements and Specifications

4.1 Performance Requirements

Angular velocity	$\omega_{req\_BG} := 1.5 \cdot \frac{deg}{s}$	[1]
	$\omega_{req\_DG} := 1.5 \cdot \frac{deg}{s}$	[1]
Angular acceleration	$\alpha_{req\_BG} := 0.3 \cdot \frac{deg}{s^2}$	[1]
	$\alpha_{req\_DG} := 0.3 \cdot \frac{deg}{s^2}$	[1]
Field diameter	$\theta_{req\_BG} := 0.2 \cdot deg$	[1]
	$\theta_{req\_DG} := 0.5 \cdot deg$	[1]
Short term time scale	$t_{shortterm} := 5 \cdot s$	[1]
Short term on-sky error (mechanical)	$\Delta\Theta_{st} := 5 \cdot mas$	[1]

4.2 Instrument Information

Rotator payload	$m_{inst\_BG} := 3500 \cdot kg$	[1]
	$m_{inst\_DG} := 3500 \cdot kg$	[1]
Maximum instrument inertia	$J_{inst\_BG} := 1750 \cdot kg \cdot m^2$	[16]
	$J_{inst\_DG} := 3940 \cdot kg \cdot m^2$	[16]
Allowable imbalance torque	$\tau_{imb\_BG} := 2000 \cdot N \cdot m$	[1]
	$\tau_{imb\_DG} := 3000 \cdot N \cdot m$	[1]

4.3 Other Information

Plate scale	$S_{BG} := 0.6000496 \cdot \frac{mm}{arcsec}$	[3]
	$S_{DG} := 0.6000496 \cdot \frac{mm}{arcsec}$	[2]
Interface to focal plane distance	$Z_{inst\_BG} := 550 \cdot mm$	[?]
	$Z_{inst\_DG} := 1500 \cdot mm$	[?]

Radius at edge of field

$$r_{\Theta\_BG} := \frac{\theta_{req\_BG}}{2} \cdot S_{BG} \qquad r_{\Theta\_BG} = 216.018\,mm$$

$$r_{\Theta\_DG} := \frac{\theta_{req\_DG}}{2} \cdot S_{DG} \qquad r_{\Theta\_DG} = 540.045\,mm$$

Allowable short term errors

Linear error at field edge

$$\Delta x_{st\_BG} := \Delta\Theta_{st} \cdot S_{BG} \qquad \Delta x_{st\_BG} = 0.003\,mm$$

$$\Delta x_{st\_DG} := \Delta\Theta_{st} \cdot S_{DG} \qquad \Delta x_{st\_DG} = 0.003\,mm$$

Angular error at field edge

$$\Delta\theta_{st\_BG} := \frac{\Delta x_{st\_BG}}{r_{\Theta\_BG}} \qquad \Delta\theta_{st\_BG} = 2.865\,arcsec$$

$$\Delta\theta_{st\_DG} := \frac{\Delta x_{st\_DG}}{r_{\Theta\_DG}} \qquad \Delta\theta_{st\_DG} = 1.146\,arcsec$$

5.0 Bearings

5.1 Bent Gregorian Rotator

Bearing pitch diameter	$d_{brg\_BG} := 1680 \cdot mm$		[5]
Bearing radial runout	$\Delta x_{brg\_BG} := 0.0006 \cdot in$		[8]
Bearing friction torque	$\tau_{brg\_BG} := 285 \cdot ft \cdot lbf$	$\tau_{brg\_BG} = 386.408\, N \cdot m$	[8]

No information was available regarding the torque specification for the other bearings. But the friction torque can be estimated from the friction torque of this bearing if it is assumed that the friction force per unit length along the pitch diameter of the bearing is constant. This is considered a reasonable assumption since the bearings are of similar construction and have the same kinds of seals.

Pitch circumference of BGR bearing	$C_{brg\_BG} := \pi \cdot d_{brg\_BG}$	$C_{brg\_BG} = 5277.876\, mn$	
Friction force per unit circumference	$f_{brg} := \dfrac{\tau_{brg\_BG}}{\dfrac{d_{brg\_BG}}{2} \cdot C_{brg\_BG}}$	$f_{brg} = 0.087\, \dfrac{N}{mm}$	
Allowable bearing moment	$M_{brg\_BG} := 20000 \cdot N \cdot m$		[1]
Bearing radial stiffness	$k_{r\_rim\_BG} := 3.38 \cdot 10^{9} \cdot \dfrac{N}{m}$		[8]
Bearing axial stiffness	$k_{a\_rim\_BG} := 4.29 \cdot 10^{9} \cdot \dfrac{N}{m}$		[8]
Bearing moment stiffness	$k_{m\_rim\_BG} := 1.27 \cdot 10^{9} \cdot \dfrac{N \cdot m}{rad}$		[8]

5.2 Bent Gregorian Cable Chain

Bearing pitch diameter	$d_{brg\_BGcc} := 1983 \cdot mm$		[4]
Bearing pitch circumference	$C_{brg\_BGcc} := \pi \cdot d_{brg\_BGcc}$		
Bearing friction torque	$\tau_{brg\_BGcc} := f_{brg} \cdot \dfrac{d_{brg\_BGcc}}{2} \cdot C_{brg\_BGcc}$	$\tau_{brg\_BGcc} = 538.36\, N \cdot m$	

5.3 Direct Gregorian Rotator

Bearing pitch diameter	$d_{brg\_DG} := 2925 \cdot mm$		[6]
Bearing pitch circumference	$C_{brg\_DG} := \pi \cdot d_{brg\_DG}$		
Bearing radial runout (this is assumed to be the same as the BGR bearing)	$\Delta x_{brg\_DG} := 0.0006 \cdot in$		[8]
Bearing friction torque (estimated)	$\tau_{brg\_DG} := f_{brg} \cdot \frac{d_{brg\_DG}}{2} \cdot C_{brg\_DG}$	$\tau_{brg\_DG} = 1171.33\, N \cdot m$	
Allowable bearing moment	$M_{brg\_DG} := 35000 \cdot N \cdot m$		[1]
Bearing radial stiffness (these are assumed to be the same as the BGR bearing)	$k_{r\_rim\_DG} := 3.38 \cdot 10^{9} \cdot \frac{N}{m}$		[8]
Bearing axial stiffness	$k_{a\_rim\_DG} := 4.29 \cdot 10^{9} \cdot \frac{N}{m}$		[8]
Bearing moment stiffness	$k_{m\_rim\_DG} := 1.27 \cdot 10^{9} \cdot \frac{N \cdot m}{rad}$		[8]

5.4 Direct Gregorian Cable Chain

Bearing pitch diameter	$d_{brg\_DGcc} := 3753 \cdot mm$		[7]
Bearing pitch circumference	$C_{brg\_DGcc} := \pi \cdot d_{brg\_DGcc}$	$C_{brg\_DGcc} = 1.179 \times 10^{4}\, mm$	
Bearing friction torque	$\tau_{brg\_DGcc} := f_{brg} \cdot \frac{d_{brg\_DGcc}}{2} \cdot C_{brg\_DGcc}$	$\tau_{brg\_DGcc} = 1928.345\, N \cdot m$	

6.0 Gears

6.1 Bent Gregorian Rotator

6.1.1 Common System Information

Pressure angle	$\phi_{gear\_BG} := 20 \cdot deg$		[5]
Module	$m_{gear\_BG} := 4 \cdot mm$		[5]
Diametral pitch	$P_{d\_rim\_BG} := \dfrac{1}{m_{gear\_BG}}$	$P_{d\_rim\_BG} = 250\,m^{-1}$	
Addendum	$a_{gear\_BG} := m_{gear\_BG}$	$a_{gear\_BG} = 4\,mm$	
Dedendum	$b_{gear\_BG} := 1.25 \cdot m_{gear\_BG}$	$b_{gear\_BG} = 5\,mm$	
Tooth hieght	$l_{gear\_BG} := a_{gear\_BG} + b_{gear\_BG}$	$l_{gear\_BG} = 9\,mm$	
Tooth thickness at pitch circle	$T_{gear\_BG} := \dfrac{\pi}{2} \cdot m_{gear\_BG}$	$T_{gear\_BG} = 6.283\,mm$	

6.1.2 Rim Gear Information

Number of teeth	$N_{rim\_BG} := 470$		[5]
Pitch diameter	$d_{p\_rim\_BG} := N_{rim\_BG} \cdot m_{gear\_BG}$	$d_{p\_rim\_BG} = 1880\,mm$	
Material hardness	$H_{B\_rim\_BG} := 248$		[5]

6.1.3 Pinion Information

Material Properties

Young's modulus	$E_{shaft\_BG} := 207 \cdot 10^{9} Pa$	[9,13]
Poisson's ratio	$\nu_{shaft\_BG} := 0.3$	[9,13]
Shear modulus	$G_{shaft\_BG} := \dfrac{E_{shaft\_BG}}{2\left(1 + \nu_{shaft\_BG}\right)}$	

Gear Information

Pinion face width $w_{pin\_BG} := 43 \cdot mm$ [9]

Number of teeth $N_{pin\_BG} := 20$ [9]

Pitch diameter $d_{p\_pin\_BG} := \dfrac{N_{pin\_BG}}{P_{d\_rim\_BG}}$ $d_{p\_pin\_BG} = 80\,mm$

Shaft Information

Diameter $d_{shaft\_BG} := 50 \cdot mm$ [9]

Length $L_{shaft\_BG} := 134 \cdot mm$ [9]

Moment of inertia for bending $I_{shaft\_BG} := \dfrac{\pi \cdot d_{shaft\_BG}^{\;4}}{64}$ $I_{shaft\_BG} = 30.68\,cm^4$

Moment of inertia for torsion $J_{shaft\_BG} := \dfrac{\pi \cdot d_{shaft\_BG}^{\;4}}{32}$ $J_{shaft\_BG} = 61.359\,cm^4$

6.2 Bent Gregorian Cable Chain

6.2.1 Common System Information

Pressure angle $\phi_{gear\_BGcc} := 20 \cdot deg$ [4]

Module $m_{gear\_BGcc} := 6 \cdot mm$ [4]

Diametral pitch $P_{d\_rim\_BGcc} := \dfrac{1}{m_{gear\_BGcc}}$ $P_{d\_rim\_BGcc} = 166.667\,m^{-1}$

Addendum $a_{gear\_BGcc} := m_{gear\_BGcc}$ $a_{gear\_BGcc} = 6\,mm$

Dedendum $b_{gear\_BGcc} := 1.25 \cdot m_{gear\_BGcc}$ $b_{gear\_BGcc} = 7.5\,mm$

Tooth hieght $l_{gear\_BGcc} := a_{gear\_BGcc} + b_{gear\_BGcc}$ $l_{gear\_BGcc} = 13.5\,mm$

Tooth thickness at pitch circle $T_{gear\_BGcc} := \dfrac{\pi}{2} \cdot m_{gear\_BGcc}$ $T_{gear\_BGcc} = 9.425\,mm$

6.2.2 Rim Gear Information

Number of teeth $N_{rim\_BGcc} := 356$ [4]

Pitch diameter $d_{p\_rim\_BGcc} := N_{rim\_BGcc} \cdot m_{gear\_BGcc}$ $d_{p\_rim\_BGcc} = 2136\,mm$

Material hardness $H_{B\_rim\_BGcc} := 248$ [4]

6.2.3 Pinion Information

Material Properties

Young's modulus $E_{shaft\_BGcc} := 207 \cdot 10^{9} Pa$ [9,13]

Poisson's ratio $\nu_{shaft\_BGcc} := 0.3$ [9,13]

Shear modulus $G_{shaft\_BGcc} := \dfrac{E_{shaft\_BGcc}}{2\left(1 + \nu_{shaft\_BGcc}\right)}$

Gear Information

Pinion face width $w_{pin\_BGcc} := 30 \cdot mm$ [9]

Number of teeth $N_{pin\_BGcc} := 16$ [9]

Pitch diameter $d_{p\_pin\_BGcc} := \dfrac{N_{pin\_BGcc}}{P_{d\_rim\_BGcc}}$ $d_{p\_pin\_BGcc} = 96\,mm$

Shaft Information

Diameter $d_{shaft\_BGcc} := 60 \cdot mm$ [9]

Length $L_{shaft\_BGcc} := 150 \cdot mm$ [9]

Moment of inertia for bending $I_{shaft\_BGcc} := \dfrac{\pi \cdot d_{shaft\_BG}{}^{4}}{64}$ $I_{shaft\_BGcc} = 30.68\,cm^{4}$

Moment of inertia for torsion $J_{shaft\_BGcc} := \dfrac{\pi \cdot d_{shaft\_BGcc}{}^{4}}{32}$ $J_{shaft\_BGcc} = 127.235\,cm^{4}$

6.3 Direct Gregorian Rotator

6.3.1 Common System Information

Pressure angle	$\phi_{gear\_DG} := 20 \cdot deg$		[5]
Module	$m_{gear\_DG} := 6 \cdot mm$		[5]
Diametral pitch	$P_{d\_rim\_DG} := \dfrac{1}{m_{gear\_DG}}$	$P_{d\_rim\_DG} = 166.667\, m^{-1}$	
Addendum	$a_{gear\_DG} := m_{gear\_DG}$	$a_{gear\_DG} = 6\, mm$	
Dedendum	$b_{gear\_DG} := 1.25 \cdot m_{gear\_DG}$	$b_{gear\_DG} = 7.5\, mm$	
Tooth hieght	$l_{gear\_DG} := a_{gear\_DG} + b_{gear\_DG}$	$l_{gear\_DG} = 13.5\, mm$	
Tooth thickness at pitch circle	$T_{gear\_DG} := \dfrac{\pi}{2} \cdot m_{gear\_DG}$	$T_{gear\_DG} = 9.425\, mm$	

6.3.2 Rim Gear Information

Number of teeth	$N_{rim\_DG} := 550$		[11]
Pitch diameter	$d_{p\_rim\_DG} := N_{rim\_DG} \cdot m_{gear\_DG}$	$d_{p\_rim\_DG} = 3300\, mm$	
Material hardness	$H_{B\_rim\_DG} := 248$		[11]

6.3.3 Pinion Information

Material Properties

Young's modulus	$E_{shaft\_DG} := 207 \cdot 10^{9} Pa$	[9,13]
Poisson's ratio	$\nu_{shaft\_DG} := 0.3$	[9,13]
Shear modulus	$G_{shaft\_DG} := \dfrac{E_{shaft\_DG}}{2\left(1 + \nu_{shaft\_DG}\right)}$	

Gear Information

Pinion face width	$w_{pin\_DG} := 43 \cdot mm$	[9]

Number of teeth $N_{pin\_DG} := 22$ [9]

Pitch diameter $d_{p\_pin\_DG} := \frac{N_{pin\_DG}}{P_{d\_rim\_DG}}$ $d_{p\_pin\_DG} = 132\,mm$

Shaft Information

Diameter $d_{shaft\_DG} := 50 \cdot mm$ [9]

Length $L_{shaft\_DG} := 134 \cdot mm$ [9]

Moment of inertia for bending $I_{shaft\_DG} := \frac{\pi \cdot d_{shaft\_DG}^{4}}{64}$ $I_{shaft\_DG} = 30.68\,cm^4$

Moment of inertia for torsion $J_{shaft\_DG} := \frac{\pi \cdot d_{shaft\_DG}^{4}}{32}$ $J_{shaft\_DG} = 61.359\,cm^4$

6.4 Direct Gregorian Cable Chain

6.4.1 Common System Information

Pressure angle $\phi_{gear\_DGcc} := 20 \cdot deg$ [7]

Module $m_{gear\_DGcc} := 6 \cdot mm$ [7]

Diametral pitch $P_{d\_rim\_DGcc} := \frac{1}{m_{gear\_DGcc}}$ $P_{d\_rim\_DGcc} = 166.667\,m^{-1}$

Addendum $a_{gear\_DGcc} := m_{gear\_DGcc}$ $a_{gear\_DGcc} = 6\,mm$

Dedendum $b_{gear\_DGcc} := 1.25 \cdot m_{gear\_DGcc}$ $b_{gear\_DGcc} = 7.5\,mm$

Tooth hieght $l_{gear\_DGcc} := a_{gear\_DGcc} + b_{gear\_DGcc}$ $l_{gear\_DGcc} = 13.5\,mm$

Tooth thickness at pitch circle $T_{gear\_DGcc} := \frac{\pi}{2} \cdot m_{gear\_DGcc}$ $T_{gear\_DGcc} = 9.425\,mm$

6.4.2 Rim Gear Information

Number of teeth $N_{rim\_DGcc} := 600$ [7]

Pitch diameter $d_{p\_rim\_DGcc} := N_{rim\_DGcc} \cdot m_{gear\_DGcc}$ $d_{p\_rim\_DGcc} = 3600\,mm$

Material hardness $H_{B\_rim\_DGcc} := 248$ [7]

6.4.3 Pinion Information

Material Properties

Young's modulus $E_{shaft\_DGcc} := 207 \cdot 10^{9} Pa$ [9,13]

Poisson's ratio $\nu_{shaft\_DGcc} := 0.3$ [9,13]

Shear modulus $G_{shaft\_DGcc} := \dfrac{E_{shaft\_DGcc}}{2\left(1 + \nu_{shaft\_DGcc}\right)}$

Gear Information

Pinion face width $w_{pin\_DGcc} := 30 \cdot mm$ [9]

Number of teeth $N_{pin\_DGcc} := 22$ [9]

Pitch diameter $d_{p\_pin\_DGcc} := \dfrac{N_{pin\_DGcc}}{P_{d\_rim\_DGcc}}$ $d_{p\_pin\_DGcc} = 132\, mm$

Shaft Information

Diameter $d_{shaft\_DGcc} := 60 \cdot mm$ [9]

Length $L_{shaft\_DGcc} := 150 \cdot mm$ [9]

Moment of inertia for bending $I_{shaft\_DGcc} := \dfrac{\pi \cdot d_{shaft\_BG}^{\,4}}{64}$ $I_{shaft\_DGcc} = 30.68\, cm^{4}$

Moment of inertia for torsion $J_{shaft\_DGcc} := \dfrac{\pi \cdot d_{shaft\_BGcc}^{\,4}}{32}$ $J_{shaft\_DGcc} = 127.235\, cm^{4}$

7.0 Motors and Mounts

7.1 Bent Gregorian Rotator

7.1.1 Torque Requirements

Acceleration torque $\tau_{acc\_BG} := J_{inst\_BG} \cdot \alpha_{req\_BG}$ $\tau_{acc\_BG} = 9.163\ N\cdot m$

Friction torque allowance $\tau_{f\_BG} := 2 \cdot \tau_{brg\_BG}$ $\tau_{f\_BG} = 772.816\ N\cdot m$

Excess drive torque allaowance $TF_{BG} := 1.25$ [16]

Total required axis torque $\tau_{ax\_max\_BG} := TF_{BG} \cdot \left(\tau_{acc\_BG} + \tau_{f\_BG}\right) + \tau_{imb\_BG}$ $\tau_{ax\_max\_BG} = 2.977\ kN\cdot m$

Reduction ratio $N_{red\_BG} := \dfrac{N_{rim\_BG}}{N_{pin\_BG}}$ $N_{red\_BG} = 23.5$

Number of motors $N_{mtr\_BG} := 2$ [9]

Torque required per motor $\tau_{mtr\_max\_BG} := \dfrac{\tau_{ax\_max\_BG}}{N_{red\_BG} \cdot N_{mtr\_BG}}$ $\tau_{mtr\_max\_BG} = 63.351\ N\cdot m$

7.1.2 Stiffness Calculation

Tangential force on pinion $F_{t\_pin\_BG} := \dfrac{2 \cdot \tau_{mtr\_max\_BG}}{d_{p\_pin\_BG}}$ $F_{t\_pin\_BG} = 1.584\ kN$

Radial force on pinion $F_{r\_pin\_BG} := F_{t\_pin\_BG} \cdot \tan\left(\phi_{gear\_BG}\right)$ $F_{r\_pin\_BG} = 0.576\ kN$

Pinion shaft deflection (separation) $\delta_{pin\_s\_BG} := \dfrac{F_{r\_pin\_BG} \cdot L_{shaft\_BG}^{3}}{3 \cdot E_{shaft\_BG} \cdot I_{shaft\_BG}}$ $\delta_{pin\_s\_BG} = 7.28\ micron$

Pinion shaft deflection (along gear) $\delta_{pin\_t\_BG} := \dfrac{F_{t\_pin\_BG} \cdot L_{shaft\_BG}^{3}}{3 \cdot E_{shaft\_BG} \cdot I_{shaft\_BG}}$ $\delta_{pin\_t\_BG} = 20.002\ micron$

Rim gear deflection (along gear) $\delta_{rim\_t\_BG} := \dfrac{F_{t\_pin\_BG}}{k_{r\_rim\_BG}}$ $\delta_{rim\_t\_BG} = 0.469\ micron$

Total deflection along gear $\delta_{tot\_BG} := \delta_{rim\_t\_BG} + \delta_{pin\_t\_BG}$ $\delta_{tot\_BG} = 20.47\ micron$

Apparent rotation of pinion $\Delta\theta_{f\_pin\_BG} := N_{red\_BG} \dfrac{\delta_{tot\_BG}}{d_{p\_rim\_BG}}$ $\Delta\theta_{f\_pin\_BG} = 52.778\ arcsec$

Torsional rotation of pinion gear $\Delta\theta_{t\_pin\_BG} := \dfrac{\tau_{mtr\_max\_BG} \cdot L_{shaft\_BG}}{J_{shaft\_BG} \cdot G_{shaft\_BG}}$ $\Delta\theta_{t\_pin\_BG} = 35.843\ \text{arcsec}$

Total rotation of pinion $\Delta\theta_{pin\_BG} := \Delta\theta_{t\_pin\_BG} + \Delta\theta_{f\_pin\_BG}$ $\Delta\theta_{pin\_BG} = 88.621\ \text{arcsec}$

Stiffness of pinion $k_{pin\_BG} := \dfrac{\tau_{mtr\_max\_BG}}{\Delta\theta_{pin\_BG}}$ $k_{pin\_BG} = 147.448\ \dfrac{kN\cdot m}{rad}$

Required drive stiffness (10x the value used in servo simulations) $k_{drv\_req\_BG} := 62600 \cdot \dfrac{N\cdot m}{rad}$ [16]

Required motor mount stiffness $k_{mnt\_req\_BG} := \dfrac{1}{\dfrac{1}{k_{drv\_req\_BG}} - \dfrac{1}{k_{pin\_BG}}}$ $k_{mnt\_req\_BG} = 108.786\ \dfrac{kN\cdot m}{rad}$

Actual mount stiffness $k_{tangent\_mnt\_BG} := 167.1 \cdot \dfrac{N}{micron}$ [14]

Actual mount stiffness at pinion $k_{mnt\_BG} := k_{tangent\_mnt\_BG} \cdot \left(\dfrac{d_{p\_rim\_BG}}{2}\right)^2$ $k_{mnt\_BG} = 147.65\ \dfrac{MN\cdot m}{rad}$

Drive stiffness (structure to ring gear) $k_{drv\_BG} := \dfrac{1}{\dfrac{1}{k_{pin\_BG}} + \dfrac{1}{k_{mnt\_BG}}}$ $k_{drv\_BG} = 1.473 \times 10^5\ \dfrac{N\cdot m}{rad}$

Inertia carried by each motor $J_{mtr\_BG} := \dfrac{J_{inst\_BG}}{N_{mtr\_BG} \cdot N_{red\_BG}^2}$

Calculated natural frequency $\omega_{drv\_BG} := \sqrt{\dfrac{k_{drv\_BG}}{J_{mtr\_BG}}}$ $\omega_{drv\_BG} = 304.906\ \dfrac{rad}{sec}$

$f_{drv\_BG} := \dfrac{\omega_{drv\_BG}}{2\cdot\pi}$ $f_{drv\_BG} = 48.527\ \text{Hz}$

7.1.3 Shaft Connection

Number of bolts $N_{B\_rot\_mtr\_BG} := 8$ [12]

Bolt circle diameter $d_{BC\_rot\_mtr\_BG} := 130 \cdot mm$ [12]

Diameter of bolts $d_{bolt\_rot\_mtr\_BG} := 8 \cdot mm$ [12]

Peak torque available $\tau_{pk\_rot\_mtr\_BG} := 501 \cdot N \cdot m$ [12]

Sliding frictional force per bolt $f_{bolt\_rot\_mtr\_BG} := \dfrac{\tau_{pk\_rot\_mtr\_BG}}{N_{B\_rot\_mtr\_BG} \cdot \dfrac{d_{BC\_rot\_mtr\_BG}}{2}}$ $f_{bolt\_rot\_mtr\_BG} = 963.462\ N$

Bolt joint friction coefficient $\mu_{bolt} := 0.15$ [13]

Required normal (bolt tension) force $T_{bolt\_rot\_mtr\_BG} := \dfrac{f_{bolt\_rot\_mtr\_BG}}{\mu_{bolt}}$ $T_{bolt\_rot\_mtr\_BG} = 6423.077\ N$

Bolt tightening torque $\tau_{bolt\_rot\_mtr\_BG} := 0.2 \cdot T_{bolt\_rot\_mtr\_BG} \cdot d_{bolt\_rot\_mtr\_BG}$ $\tau_{bolt\_rot\_mtr\_BG} = 10.277\ N \cdot m$

$\tau_{bolt\_rot\_mtr\_BG} = 7.58\ ft \cdot lbf$

7.2 Bent Gregorian Cable Chain

7.2.1 Torque Requirements

Total required axis torque $\tau_{axis\_max\_BGcc} := 1782 \cdot N \cdot m$ [18]

Gearbox reduction ratio $N_{GB\_BGcc} := 10$ [9]

Total Reduction ratio $N_{red\_BGcc} := \dfrac{N_{GB\_BGcc} \cdot N_{rim\_BGcc}}{N_{pin\_BGcc}}$ $N_{red\_BGcc} = 222.5$

Number of motors $N_{mtr\_BGcc} := 1$ [9]

Torque required per motor $\tau_{mtr\_max\_BGcc} := \dfrac{\tau_{axis\_max\_BGcc}}{N_{red\_BGcc} \cdot N_{mtr\_BGcc}}$ $\tau_{mtr\_max\_BGcc} = 8.009\ N \cdot m$

Gearbox output torque $\tau_{GB\_max\_BGcc} := \tau_{mtr\_max\_BGcc} \cdot N_{GB\_BGcc}$ $\tau_{GB\_max\_BGcc} = 80.09\ N \cdot m$

$\tau_{GB\_max\_BGcc} = 708.855\ in \cdot lbf$

Tangential force on pinion $F_{t\_pin\_BGcc} := \dfrac{2 \cdot \tau_{GB\_max\_BGcc}}{d_{p\_pin\_BGcc}}$ $F_{t\_pin\_BGcc} = 1.669\ kN$

Radial force on pinion $F_{r\_pin\_BGcc} := F_{t\_pin\_BGcc} \cdot \tan\left(\phi_{gear\_BGcc}\right)$ $F_{r\_pin\_BGcc} = 0.607\ kN$

7.2.2 Shaft Connection

Number of bolts $N_{B\_BGcc\_mtr} := 8$ [12]

Bolt circle diameter $d_{BC\_BGcc\_mtr} := 80 \cdot mm$ [12]

Diameter of bolts $d_{bolt\_BGcc\_mtr} := 6 \cdot mm$ [12]

Peak torque available $\tau_{pk\_BGcc\_mtr} := 64.4 \cdot N \cdot m$ [12]

Sliding frictional force per bolt $f_{bolt\_BGcc\_mtr} := \dfrac{\tau_{pk\_BGcc\_mtr}}{N_{B\_BGcc\_mtr} \cdot \dfrac{d_{BC\_BGcc\_mtr}}{2}}$ $f_{bolt\_BGcc\_mtr} = 201.25\ N$

Required normal (bolt tension) force $T_{bolt\_BGcc\_mtr} := \dfrac{f_{bolt\_BGcc\_mtr}}{\mu_{bolt}}$ $T_{bolt\_BGcc\_mtr} = 1341.667\ N$

Bolt tightening torque $\tau_{bolt\_BGcc\_mtr} := 0.2 \cdot T_{bolt\_BGcc\_mtr} \cdot d_{bolt\_BGcc\_mtr}$ $\tau_{bolt\_BGcc\_mtr} = 1.61\ N \cdot m$

$\tau_{bolt\_BGcc\_mtr} = 1.187\ ft \cdot lbf$

7.3 Direct Gregorian Rotator

7.3.1 Torque Requirements

Acceleration torque $\tau_{acc\_DG} := J_{inst\_DG} \cdot \alpha_{req\_DG}$ $\tau_{acc\_DG} = 20.63\ N \cdot m$

Friction torque allowance $\tau_{f\_DG} := 2 \cdot \tau_{brg\_DG}$ $\tau_{f\_DG} = 2342.661\ N \cdot m$

Excess drive torque allaowance $TF_{DG} := 1.25$ [16]

Total required axis torque $\tau_{ax\_max\_DG} := TF_{DG} \cdot \left(\tau_{acc\_DG} + \tau_{f\_DG}\right) + \tau_{imb\_DG}$ $\tau_{ax\_max\_DG} = 5.954\ kN \cdot m$

Reduction ratio $N_{red\_DG} := \dfrac{N_{rim\_DG}}{N_{pin\_DG}}$ $N_{red\_DG} = 25$

Number of motors $N_{mtr\_DG} := 2$ [9]

Torque required per motor $\tau_{mtr\_max\_DG} := \dfrac{\tau_{ax\_max\_DG}}{N_{red\_DG} \cdot N_{mtr\_DG}}$ $\tau_{mtr\_max\_DG} = 119.082\ N \cdot m$

7.3.2 Stiffness Calculation

Tangential force on pinion	$F_{t\_pin\_DG} := \dfrac{2 \cdot \tau_{mtr\_max\_DG}}{d_{p\_pin\_DG}}$	$F_{t\_pin\_DG} = 1.804\,kN$
Radial force on pinion	$F_{r\_pin\_DG} := F_{t\_pin\_DG} \cdot \tan\left(\phi_{gear\_DG}\right)$	$F_{r\_pin\_DG} = 0.657\,kN$
Pinion shaft deflection (separation)	$\delta_{pin\_s\_DG} := \dfrac{F_{r\_pin\_DG} \cdot L_{shaft\_DG}{}^{3}}{3 \cdot E_{shaft\_DG} \cdot I_{shaft\_DG}}$	$\delta_{pin\_s\_DG} = 8.294\,micron$
Pinion shaft deflection (along gear)	$\delta_{pin\_t\_DG} := \dfrac{F_{t\_pin\_DG} \cdot L_{shaft\_DG}{}^{3}}{3 \cdot E_{shaft\_DG} \cdot I_{shaft\_DG}}$	$\delta_{pin\_t\_DG} = 22.786\,micron$
Rim gear deflection (along gear)	$\delta_{rim\_t\_DG} := \dfrac{F_{t\_pin\_DG}}{k_{r\_rim\_DG}}$	$\delta_{rim\_t\_DG} = 0.534\,micron$
Total deflection along gear	$\delta_{tot\_DG} := \delta_{rim\_t\_DG} + \delta_{pin\_t\_DG}$	$\delta_{tot\_DG} = 23.32\,micron$
Apparent rotation of pinion	$\Delta\theta_{f\_pin\_DG} := N_{red\_DG} \dfrac{\delta_{tot\_DG}}{d_{p\_rim\_DG}}$	$\Delta\theta_{f\_pin\_DG} = 36.44\,arcsec$
Torsional rotation of pinion gear	$\Delta\theta_{t\_pin\_DG} := \dfrac{\tau_{mtr\_max\_DG} \cdot L_{shaft\_DG}}{J_{shaft\_DG} \cdot G_{shaft\_DG}}$	$\Delta\theta_{t\_pin\_DG} = 67.375\,arcse$
Total rotation of pinion	$\Delta\theta_{pin\_DG} := \Delta\theta_{t\_pin\_DG} + \Delta\theta_{f\_pin\_DG}$	$\Delta\theta_{pin\_DG} = 103.816\,arcsec$
Stiffness of pinion	$k_{pin\_DG} := \dfrac{\tau_{mtr\_max\_DG}}{\Delta\theta_{pin\_DG}}$	$k_{pin\_DG} = 236.597\,\dfrac{kN \cdot m}{rad}$
Required drive stiffness (10x the value used in servo simulations)	$k_{drv\_req\_DG} := 124000 \cdot N \cdot \dfrac{m}{rad}$	[5]
Required motor mount stiffness	$k_{mnt\_req\_DG} := \dfrac{1}{\dfrac{1}{k_{drv\_req\_DG}} - \dfrac{1}{k_{pin\_DG}}}$	$k_{mnt\_req\_DG} = 260.558\,\dfrac{kN \cdot m}{rad}$
Actual mount stiffness	$k_{tangent\_mnt\_DG} := 167.1 \cdot \dfrac{N}{micron}$	[11]

Actual mount stiffness at pinion $k_{mnt\_DG} := k_{tangent\_mnt\_DG} \cdot \left(\frac{d_{p\_rim\_DG}}{2}\right)^2$ $k_{mnt\_DG} = 454.93 \frac{MN \cdot m}{rad}$

Drive stiffness (structure to ring gear) $k_{drv\_DG} := \frac{1}{\frac{1}{k_{pin\_DG}} + \frac{1}{k_{mnt\_DG}}}$ $k_{drv\_DG} = 2.365 \times 10^5 \frac{N \cdot m}{rad}$

Inertia carried by each motor $J_{mtr\_DG} := \frac{J_{inst\_DG}}{N_{mtr\_DG} \cdot N_{red\_DG}^2}$

Calculated natural frequency $\omega_{drv\_DG} := \sqrt{\frac{k_{drv\_DG}}{J_{mtr\_DG}}}$ $\omega_{drv\_DG} = 273.904 \frac{rad}{sec}$

$f_{drv\_DG} := \frac{\omega_{drv\_BG}}{2 \cdot \pi}$ $f_{drv\_DG} = 48.527\,Hz$

7.3.3 Shaft Connection

Number of bolts $N_{B\_rot\_mtr\_DG} := 8$ [12]

Bolt circle diameter $d_{BC\_rot\_mtr\_DG} := 130 \cdot mm$ [12]

Diameter of bolts $d_{bolt\_rot\_mtr\_DG} := 8 \cdot mm$ [12]

Peak torque available $\tau_{pk\_rot\_mtr\_DG} := 501 \cdot N \cdot m$ [12]

Sliding frictional force per bolt $f_{bolt\_rot\_mtr\_DG} := \frac{\tau_{pk\_rot\_mtr\_DG}}{N_{B\_rot\_mtr\_DG} \cdot \frac{d_{BC\_rot\_mtr\_DG}}{2}}$ $f_{bolt\_rot\_mtr\_DG} = 963.462\ N$

Required normal (bolt tension) force $T_{bolt\_rot\_mtr\_DG} := \frac{f_{bolt\_rot\_mtr\_DG}}{\mu_{bolt}}$ $T_{bolt\_rot\_mtr\_DG} = 6423.077\ N$

Bolt tightening torque $\tau_{bolt\_rot\_mtr\_DG} := 0.2 \cdot T_{bolt\_rot\_mtr\_DG} \cdot d_{bolt\_rot\_mtr\_DG}$

$\tau_{bolt\_rot\_mtr\_DG} = 10.277\,N \cdot m$

$\tau_{bolt\_rot\_mtr\_DG} = 7.58\,ft \cdot lbf$

7.4 Direct Gregorian Cable Chain

7.4.1 Torque Requirements

Total required axis torque $\tau_{axis\_max\_DGcc} := 2688 \cdot N \cdot m$ [18]

Gearbox reduction ratio $N_{GB\_DGcc} := 10$ [5]

Total Reduction ratio $N_{red\_DGcc} := \dfrac{N_{GB\_DGcc} \cdot N_{rim\_DGcc}}{N_{pin\_DGcc}}$ $N_{red\_DGcc} = 272.727$

Number of motors $N_{mtr\_DGcc} := 1$ [5]

Torque required per motor $\tau_{mtr\_max\_DGcc} := \dfrac{\tau_{axis\_max\_DGcc}}{N_{red\_DGcc} \cdot N_{mtr\_DGcc}}$ $\tau_{mtr\_max\_DGcc} = 9.856\, N \cdot m$

Gearbox output torque $\tau_{GB\_max\_DGcc} := \tau_{mtr\_max\_DGcc} \cdot N_{GB\_DGcc}$ $\tau_{GB\_max\_DGcc} = 98.56\, N \cdot m$

$\tau_{GB\_max\_DGcc} = 872.33\, in \cdot lbf$

Tangential force on pinion $F_{t\_pin\_DGcc} := \dfrac{2 \cdot \tau_{GB\_max\_DGcc}}{d_{p\_pin\_DGcc}}$ $F_{t\_pin\_DGcc} = 1.493\, kN$

Radial force on pinion $F_{r\_pin\_DGcc} := F_{t\_pin\_DGcc} \cdot \tan\left(\phi_{gear\_DGcc}\right)$ $F_{r\_pin\_DGcc} = 0.544\, kN$

7.4.2 Shaft Connection

Number of bolts $N_{B\_DGcc\_mtr} := 8$ [12]

Bolt circle diameter $d_{BC\_DGcc\_mtr} := 80 \cdot mm$ [12]

Diameter of bolts $d_{bolt\_DGcc\_mtr} := 6 \cdot mm$ [12]

[12]

Peak torque available $\tau_{pk\_DGcc\_mtr} := 64.4 \cdot N \cdot m$

Sliding frictional force per bolt $f_{bolt\_DGcc\_mtr} := \dfrac{\tau_{pk\_DGcc\_mtr}}{N_{B\_DGcc\_mtr} \cdot \dfrac{d_{BC\_DGcc\_mtr}}{2}}$ $f_{bolt\_DGcc\_mtr} = 201.25\ N$

Required normal (bolt tension) force $T_{bolt\_DGcc\_mtr} := \dfrac{f_{bolt\_DGcc\_mtr}}{\mu_{bolt}}$ $T_{bolt\_DGcc\_mtr} = 1341.667\ N$

Bolt tightening torque $\tau_{bolt\_DGcc\_mtr} := 0.2 \cdot T_{bolt\_DGcc\_mtr} \cdot d_{bolt\_DGcc\_mtr}$ $\tau_{bolt\_DGcc\_mtr} = 1.61\ N \cdot m$

$\tau_{bolt\_DGcc\_mtr} = 1.187\ ft \cdot lbf$

8.0 Encoders and Mounts

8.1 General Tape Information

Tape thickness $t_{tape} := 0.008 \cdot in$ [17]

Tape width $w_{tape} := 40 \cdot mm$ [17]

Cross-sectional area $A_{tape} := w_{tape} \cdot t_{tape}$ $A_{tape} = 8.128\ mm^2$

Installation tension $T_{tape} := 40 \cdot lbf$ [17]

Youngs modulus $E_{tape} := 209 \cdot 10^9 \cdot Pa$ [13]

Stress in tape at maximum tension $\sigma_{tape} := \dfrac{T_{tape}}{A_{tape}}$ $\sigma_{tape} = 21.891\ MPa$

8.2 Bent Gregorian Rotator

Tape pitch $p_{tape\_BG} := 0.1 \cdot in$ [16]

Approximate diameter $d_{enc\_apx\_BG} := 1500 \cdot mm$ [9]

Approximate length $L_{enc\_apx\_BG} := \pi \cdot d_{enc\_apx\_BG}$ $L_{enc\_apx\_BG} = 4712.389\ mm$

Periods for approximate length $N_{enc\_apx\_BG} := \frac{L_{enc\_apx\_BG}}{p_{tape\_BG}}$ $N_{enc\_apx\_BG} = 1855.271$

Exact length $L_{enc\_BG} := p_{tape\_BG} \cdot \mathrm{trunc}\left(N_{enc\_apx\_BG}\right)$ $L_{enc\_BG} = 4711.700\,mm$

Exact diameter $d_{enc\_BG} := \frac{L_{enc\_BG}}{\pi} - t_{tape}$ $d_{enc\_BG} = 1499.577\,mm$

The diameter should be slightly larger than this calculated exact diameter to ensure there is a gap rather than an overlap. So, the tolerances should be set such that the diameter may not vary below this. The high diameter limit will be calculated such that a tension of twice the nominal tension will close the gap. The minimum tension will be set at half the nominal to ensure there is a gap at zero tension and minimum diameter.

Low diameter tolerance $\delta_{l\_BG} := 0.0 \cdot mm$

Length stretch at half tension $\Delta L_{enc\_min\_BG} := 0.5 \cdot \frac{T_{tape} \cdot L_{enc\_BG}}{A_{tape} \cdot E_{tape}}$ $\Delta L_{enc\_min\_BG} = 0.247\,mm$

Length stretch at double tension $\Delta L_{enc\_max\_BG} := 2 \cdot \frac{T_{tape} \cdot L_{enc\_BG}}{A_{tape} \cdot E_{tape}}$ $\Delta L_{enc\_max\_BG} = 0.987\,mm$

Low diameter tolerance $\delta_{l\_BG} := \frac{\Delta L_{enc\_min\_BG}}{\pi}$ $\delta_{l\_BG} = 0.079\,mm$

High diameter tolerance $\delta_{h\_BG} := \frac{\Delta L_{enc\_max\_BG}}{\pi}$ $\delta_{h\_BG} = 0.314\,mm$

Minimum surface diameter $d_{enc\_l\_BG} := d_{enc\_BG} + \delta_{l\_BG}$ $d_{enc\_l\_BG} = 1499.656\,mm$

Maximum surface diameter $d_{enc\_h\_BG} := d_{enc\_BG} + \delta_{h\_BG}$ $d_{enc\_h\_BG} = 1499.892\,mm$

8.3 Direct Gregorian Rotator

Tape pitch $p_{tape\_DG} := 0.1 \cdot in$ [16]

Approximate diameter $d_{enc\_apx\_DG} := 2900 \cdot mm$ [9]

Approximate length $L_{enc\_apx\_DG} := \pi \cdot d_{enc\_apx\_DG}$ $L_{enc\_apx\_DG} = 9110.619\,mm$

Periods for approximate length $N_{enc\_apx\_DG} := \frac{L_{enc\_apx\_DG}}{p_{tape\_DG}}$ $N_{enc\_apx\_DG} = 3586.858$

Exact length $L_{enc\_DG} := p_{tape\_DG} \cdot \mathrm{trunc}\left(N_{enc\_apx\_DG}\right)$ $L_{enc\_DG} = 9108.440\,mm$

Exact diameter $d_{enc\_DG} := \frac{L_{enc\_DG}}{\pi} - t_{tape}$ $d_{enc\_DG} = 2899.103\,mm$

The diameter tolerances are established as described above.

Low diameter tolerance $\delta_{l\_DG} := 0.0 \cdot mm$

Length stretch at half tension $\Delta L_{enc\_min\_DG} := 0.5 \cdot \frac{T_{tape} \cdot L_{enc\_DG}}{A_{tape} \cdot E_{tape}}$ $\Delta L_{enc\_min\_DG} = 0.477\,mm$

Length stretch at double tension $\Delta L_{enc\_max\_DG} := 2 \cdot \frac{T_{tape} \cdot L_{enc\_DG}}{A_{tape} \cdot E_{tape}}$ $\Delta L_{enc\_max\_DG} = 1.908\,mm$

Low diameter tolerance $\delta_{l\_DG} := \frac{\Delta L_{enc\_min\_DG}}{\pi}$ $\delta_{l\_DG} = 0.152\,mm$

High diameter tolerance $\delta_{h\_DG} := \frac{\Delta L_{enc\_max\_DG}}{\pi}$ $\delta_{h\_DG} = 0.607\,mm$

Minimum surface diameter $d_{enc\_l\_DG} := d_{enc\_DG} + \delta_{l\_DG}$ $d_{enc\_l\_DG} = 2899.255\,mm$

Maximum surface diameter $d_{enc\_h\_DG} := d_{enc\_DG} + \delta_{h\_DG}$ $d_{enc\_h\_DG} = 2899.711\,mm$

9.0 Limits and Stopping

9.1 Bent Gregorian Rotator

The position sensor acts to sense the relative position of the rotator and cable chain and also incorporates the proximity limit switches. The middle 80% of the range of teh device will be used for position control. The proximity switches will be set at 10% and 90% (+/- 40% of the range)

Radius of position sensor $r_{ps\_BG} := 785 \cdot mm$ [9]

Length of position sensor $L_{ps\_BG} := 80 \cdot mm$ [9]

Angular travel to limit $\Delta\theta_{rot\_BGcc} := 0.4 \cdot \frac{L_{ps\_BG}}{r_{ps\_BG}}$ $\Delta\theta_{rot\_BGcc} = 2.336\,deg$

Radius of hard stop $r_{hs\_BG} := 800 \cdot mm$ [9]

Travel distance $\Delta\theta_{BGcc} := 1.5 \cdot deg$ [9]

Compression distance $\Delta L_{hs\_BG} := r_{hs\_BG} \cdot \Delta\theta_{BGcc}$ $\Delta L_{hs\_BG} = 20.944\,mm$

9.2 Direct Gregorian Rotator

Radius of position sensor $r_{ps\_DG} := 1600 \cdot mm$ [9]

Length of position sensor $L_{ps\_DG} := 120 \cdot mm$ [9]

Angular travel to limit $\Delta\theta_{rot\_DGcc} := 0.4 \cdot \frac{L_{ps\_DG}}{r_{ps\_DG}}$ $\Delta\theta_{rot\_DGcc} = 1.719\,deg$

Radius of hard stop $r_{hs\_DG} := 1700 \cdot mm$ [9]

Travel distance $\Delta\theta_{DGcc} := 1.5 \cdot deg$ [9]

Compression distance $\Delta L_{hs\_DG} := r_{hs\_DG} \cdot \Delta\theta_{DGcc}$ $\Delta L_{hs\_DG} = 44.506\,m$

10.0 Gear and Shaft Stresses

10.1 Bent Gregorian Rotator

10.1.1 Operating Conditions

Bending stress in pinion shaft $\sigma_{pin\_ben\_BG} := \frac{F_{r\_pin\_BG} \cdot L_{shaft\_BG} \cdot d_{shaft\_BG}}{2 \cdot I_{shaft\_BG}}$ $\sigma_{pin\_ben\_BG} = 6.294\,MPa$

Torsional stress in pinion shaft $\tau_{pin\_tor\_BG} := \dfrac{\tau_{mtr\_max\_BG} \cdot d_{shaft\_BG}}{2 \cdot J_{shaft\_BG}}$ $\tau_{pin\_tor\_BG} = 2.581\ MPa$

Lewis form factor Y from table $Y_{BG} := 0.30769$ [13]

A smaller value of Y increases the stress. The value above is the worst case from the table in the reference for the particular number of teeth and pressure angle..

Tooth bending stress $\sigma_{bend\_BG} := \dfrac{F_{t\_pin\_BG} \cdot P_{d\_rim\_BG}}{w_{pin\_BG} \cdot Y_{BG}}$ $\sigma_{bend\_BG} = 29.926\ MPa$

$\sigma_{bend\_BG} = 4340.396\ psi$

The strength of the gear material is related to its hardness. The strength is then about

Gear strength $S_{u\_BG} := 3.45 \cdot MPa \cdot H_{B\_rim\_BG}$ $S_{u\_BG} = 855.6\ MPa$

For steel alloys, the edurance limit is about 40% of the ultimate strength

Endurance limit $S_{e\_BG} := 0.4 \cdot S_{u\_BG}$ $S_{e\_BG} = 342.24\ MP$

10.1.2 Emergency Stop Conditions

Maximum torque applied to pinion $\tau_{maxES\_BG} := 400 \cdot N \cdot m$ [15]

This torque comes from the brake because the brake capacity must always be higher than the peak available motor torque

Tangential force on pinion $F_{t\_pinES\_BG} := \dfrac{2 \cdot \tau_{maxES\_BG}}{d_{p\_pin\_BG}}$ $F_{t\_pinES\_BG} = 10\ kN$

Radial force on pinion $F_{r\_pinES\_BG} := F_{t\_pinES\_BG} \cdot \tan(\phi_{gear\_BG})$ $F_{r\_pinES\_BG} = 3.64\,kN$

Bending stress in pinion shaft $\sigma_{pin\_benES\_BG} := \dfrac{F_{r\_pinES\_BG} \cdot L_{shaft\_BG} \cdot d_{shaft\_BG}}{2 \cdot I_{shaft\_BG}}$ $\sigma_{pin\_benES\_BG} = 39.743\,MPa$

Torsional stress in pinion shaft $\tau_{pin\_torES\_BG} := \dfrac{\tau_{maxES\_BG} \cdot d_{shaft\_BG}}{2 \cdot J_{shaft\_BG}}$ $\tau_{pin\_torES\_BG} = 16.297\,MPa$

Tooth bending stress $\sigma_{bendES\_BG} := \dfrac{F_{t\_pinES\_BG} \cdot P_{d\_rim\_BG}}{w_{pin\_BG} \cdot Y_{BG}}$ $\sigma_{bendES\_BG} = 188.955\,MPa$

$\sigma_{bendES\_BG} = 2.741 \times 10^4\,ps$

10.2 Bent Gregorian Cable Chain

10.2.1 Operating Conditions

Bending stress in pinion shaft $\sigma_{pin\_ben\_BGcc} := \dfrac{F_{r\_pin\_BGcc} \cdot L_{shaft\_BGcc} \cdot d_{shaft\_BGcc}}{2 \cdot I_{shaft\_BGcc}}$ $\sigma_{pin\_ben\_BGcc} = 8.908\,MPa$

Torsional stress in pinion shaft $\tau_{pin\_tor\_BGcc} := \dfrac{\tau_{mtr\_max\_BGcc} \cdot d_{shaft\_BGcc}}{2 \cdot J_{shaft\_BGcc}}$ $\tau_{pin\_tor\_BGcc} = 0.189\,MPa$

Lewis form factor Y from table $Y_{BGcc} := 0.27610$ [13]

Tooth bending stress $\sigma_{bend\_BGcc} := \dfrac{F_{t\_pin\_BGcc} \cdot P_{d\_rim\_BGcc}}{w_{pin\_BGcc} \cdot Y_{BGcc}}$ $\sigma_{bend\_BGcc} = 33.574\,MPa$

$\sigma_{bend\_BGcc} = 4869.435\,ps$

Gear strength $S_{u\_BGcc} := 3.45 \cdot MPa \cdot H_{B\_rim\_BGcc}$ $S_{u\_BGcc} = 855.6\,MPa$

For steel alloys, the edurance limit is about 40% of the ultimate strength

Endurance limit $S_{e\_BGcc} := 0.4 \cdot S_{u\_BGcc}$ $S_{e\_BGcc} = 342.24\,MPa$

10.3 Direct Gregorian Rotator

10.3.1 Operating Conditions

Bending stress in pinion shaft $\sigma_{pin\_ben\_DG} := \dfrac{F_{r\_pin\_DG} \cdot L_{shaft\_DG} \cdot d_{shaft\_DG}}{2 \cdot I_{shaft\_DG}}$ $\sigma_{pin\_ben\_DG} = 7.171\ \mathrm{MPa}$

Torsional stress in pinion shaft $\tau_{pin\_tor\_DG} := \dfrac{\tau_{mtr\_max\_DG} \cdot d_{shaft\_DG}}{2 \cdot J_{shaft\_DG}}$ $\tau_{pin\_tor\_DG} = 4.852\ \mathrm{MPa}$

Lewis form factor Y from table $Y_{DG} := 0.31997$ [13]

A smaller value of Y increases the stress. The value above is the worst case from the table in the reference for the particular number of teeth and pressure angle..

Tooth bending stress $\sigma_{bend\_DG} := \dfrac{F_{t\_pin\_DG} \cdot P_{d\_rim\_DG}}{w_{pin\_DG} \cdot Y_{DG}}$ $\sigma_{bend\_DG} = 21.856\ \mathrm{MPa}$

$\sigma_{bend\_DG} = 3169.97\ \mathrm{psi}$

The strength of the gear material is related to its hardness. The strength is then about

Gear strength $S_{u\_DG} := 3.45 \cdot \mathrm{MPa} \cdot H_{B\_rim\_DG}$ $S_{u\_DG} = 855.6\ \mathrm{MPa}$

For steel alloys, the edurance limit is about 40% of the ultimate strength

Endurance limit $S_{e\_DG} := 0.4 \cdot S_{u\_DG}$ $S_{e\_DG} = 342.24\ \mathrm{MP}$

10.3.2 Emergency Stop Conditions

Maximum torque applied to pinion $\tau_{maxES\_DG} := 400 \cdot \mathrm{N} \cdot \mathrm{m}$ [15]

This torque comes from the brake because the brake capacity must always be higher than the peak available motor torque

Tangential force on pinion $F_{t\_pinES\_DG} := \dfrac{2 \cdot \tau_{maxES\_DG}}{d_{p\_pin\_DG}}$ $F_{t\_pinES\_DG} = 6.061\ \mathrm{kN}$

Radial force on pinion $F_{r\_pinES\_DG} := F_{t\_pinES\_DG} \cdot \tan\left(\phi_{gear\_DG}\right)$ $F_{r\_pinES\_DG} = 2.206\ \mathrm{kN}$

Bending stress in pinion shaft $\sigma_{pin\_benES\_DG} := \dfrac{F_{r\_pinES\_DG} \cdot L_{shaft\_DG} \cdot d_{shaft\_DG}}{2 \cdot I_{shaft\_DG}}$ $\sigma_{pin\_benES\_DG} = 24.087\,MPa$

Torsional stress in pinion shaft $\tau_{pin\_torES\_DG} := \dfrac{\tau_{maxES\_DG} \cdot d_{shaft\_DG}}{2 \cdot J_{shaft\_DG}}$ $\tau_{pin\_torES\_DG} = 16.297\,MPa$

Tooth bending stress $\sigma_{bendES\_DG} := \dfrac{F_{t\_pinES\_DG} \cdot P_{d\_rim\_DG}}{w_{pin\_DG} \cdot Y_{DG}}$ $\sigma_{bendES\_DG} = 73.415\,MPa$

$\sigma_{bendES\_DG} = 1.065 \times 10^{4}\,ps$

10.4 Direct Gregorian Cable Chain

10.4.1 Operating Conditions

Bending stress in pinion shaft $\sigma_{pin\_ben\_DGcc} := \dfrac{F_{r\_pin\_DGcc} \cdot L_{shaft\_DGcc} \cdot d_{shaft\_DGcc}}{2 \cdot I_{shaft\_DGcc}}$ $\sigma_{pin\_ben\_DGcc} = 7.972\,MPa$

Torsional stress in pinion shaft $\tau_{pin\_tor\_DGcc} := \dfrac{\tau_{mtr\_max\_DGcc} \cdot d_{shaft\_DGcc}}{2 \cdot J_{shaft\_DGcc}}$ $\tau_{pin\_tor\_DGcc} = 0.232\,MPa$

Lewis form factor Y from table $Y_{DGcc} := 0.31997$ [13]

Tooth bending stress $\sigma_{bend\_DGcc} := \dfrac{F_{t\_pin\_DGcc} \cdot P_{d\_rim\_DGcc}}{w_{pin\_DGcc} \cdot Y_{DGcc}}$ $\sigma_{bend\_DGcc} = 25.928\,MPa$

$\sigma_{bend\_DGcc} = 3760.59\,psi$

Gear strength $S_{u\_DGcc} := 3.45 \cdot MPa \cdot H_{B\_rim\_DGcc}$ $S_{u\_DGcc} = 855.6\,MPa$

For steel alloys, the edurance limit is about 40% of the ultimate strength

Endurance limit $S_{e\_DGcc} := 0.4 \cdot S_{u\_DGcc}$ $S_{e\_DGcc} = 342.24\,MPa$

11.0 On-Sky Mechanical Errors

The short term on-sky error contributed by mechanical effects come from two sources - ball diameter errors and bearing runout errors. To be conservative we can assume that ball errors act directly to move the instrument around the sky, i. e., there is no averaging. Similarly, we can assume that race errors occur over a 90 degree section.

11.1 Bent Gregorian Rotator

Ball diameter error $\Delta d_{ball} := 24 \cdot 10^{-6} \cdot in$ [15]

Angle turned in a "short term" $\theta_{shortterm} := \omega_{req\_BG} \cdot t_{shortterm}$ $\theta_{shortterm} = 7.5\,deg$

Random tilt error $\Delta\alpha_{ball\_BG} := \dfrac{\Delta d_{ball}}{d_{brg\_BG}}$

Random on-sky error $\Delta p_{ball\_BG} := \dfrac{\Delta d_{ball}}{S_{BG}}$ $\Delta p_{ball\_BG} = 1.016\,mas$

Short term on-sky error from race $\Delta p_{race\_BG} := \dfrac{\theta_{shortterm}}{90 \cdot deg} \cdot \dfrac{\Delta x_{brg\_BG}}{S_{BG}}$ $\Delta p_{race\_BG} = 2.116\,mas$

Total on-sky mechanical error $\Delta p_{sky\_BG} := \sqrt{\Delta p_{ball\_BG}^{\,2} + \Delta p_{race\_BG}^{\,2}}$ $\Delta p_{sky\_BG} = 2.348\,mas$

11.2 Direct Gregorian Rotator

Random tilt error $\Delta\alpha_{ball\_DG} := \dfrac{\Delta d_{ball}}{d_{brg\_DG}}$

Random on-sky error $\Delta p_{ball\_DG} := \dfrac{\Delta d_{ball}}{S_{DG}}$ $\Delta p_{ball\_DG} = 1.016\,mas$

Short term on-sky error from race $\Delta p_{race\_DG} := \dfrac{\theta_{shortterm}}{90 \cdot deg} \cdot \dfrac{\Delta x_{brg\_DG}}{S_{DG}}$ $\Delta p_{race\_DG} = 2.116\,mas$

Total on-sky mechanical error $\Delta p_{sky\_DG} := \sqrt{\Delta p_{ball\_DG}^{2} + \Delta p_{race\_DG}^{2}}$ $\Delta p_{sky\_DG} = 2.348\,\text{mas}$

12.0 Misalignment with Elevation Angle

12.1 Bent Gregorian Rotator

Decenter $\Delta y_{EL\_BG} := \dfrac{m_{inst\_BG} \cdot g}{k_{r\_rim\_BG}}$ $\Delta y_{EL\_BG} = 10.155\,\text{micron}$

Pointing error induced by decenter $\Delta\Theta_{dc\_BG} := \dfrac{\Delta y_{EL\_BG}}{S_{BG}}$ $\Delta\Theta_{dc\_BG} = 0.017\,\text{arcsec}$

Defocus $\Delta z_{EL\_BG} := \dfrac{m_{inst\_BG} \cdot g}{k_{a\_rim\_BG}}$ $\Delta z_{EL\_BG} = 8.001\,\text{micron}$

Tilt $\Delta\psi_{EL\_BG} := \dfrac{m_{inst\_BG} \cdot g \cdot \dfrac{Z_{inst\_BG}}{2}}{k_{m\_rim\_BG}}$ $\Delta\psi_{EL\_BG} = 1.533\,\text{arcsec}$

12.2 Direct Gregorian Rotator

Decenter $\Delta y_{EL\_DG} := \dfrac{m_{inst\_DG} \cdot g}{k_{r\_rim\_DG}}$ $\Delta y_{EL\_DG} = 10.155\,\text{micron}$

Pointing error induced by decenter $\Delta\Theta_{dc\_DG} := \dfrac{\Delta y_{EL\_DG}}{S_{DG}}$ $\Delta\Theta_{dc\_DG} = 0.017\,\text{arcsec}$

Defocus $\Delta z_{EL\_DG} := \dfrac{m_{inst\_DG} \cdot g}{k_{a\_rim\_DG}}$ $\Delta z_{EL\_DG} = 8.001\,\text{micron}$

Tilt

$$\Delta\psi_{EL\_DG} := \frac{m_{inst\_DG} \cdot g \cdot \frac{Z_{inst\_DG}}{2}}{k_{m\_rim\_DG}}$$

$$\Delta\psi_{EL\_DG} = 4.181\ \text{arcsec}$$

Auxiliary Unit Definitions

$MN \equiv 10^{6} \cdot N$ $\qquad$ $mas \equiv 10^{-3} \cdot arcsec$